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Empirical Performance of a Model-Free Volatility against the Different Option Strike Size Discreteness

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ABSTRACT

This study investigates whether the different step size of option strike price discreteness contributes to the performance of a model-free variance in approximating the real-value volatility. The volatility is proxied by the volatility implied by the Black-Scholes-Merton option pricing model. We concentrate on examining the respective relationship governing the function of approximation error against the strike price step. A sample data extracted from DJIA index options data is used, which covers the period from January 2009 until the end of 2015. This study finds that the best strike price step size that asserts the most minimum approximation error by practice is a step size of $1.00. There exists a linear relationship between strike price discreteness size and approximation error. The choice of the different step size of strike price discreteness is in fact contributes to the performance of a model-free variance in approximating the real-value volatility.

Keywords: model-free, options, strike price discreteness and volatility.

1. Introduction

Option pricing has indeed remained practical either by its theory per se or its application. Owing to this fact, a large number of researchers tend to shed their light by focusing on this realm of work. The attention has been phenomenal especially since the extensive study by Black and Scholes (1973) in developing option pricing model. The model introduced by Black and Scholes (1973) and Merton (1973), i.e. the Black-Scholes-Merton (BSM) model has been acknowledged as a standard paradigm in finance realm and is most extensively used model, despite its failure to hold several assumptions. This has sparked a plethora of study on the option-implied subject. This research attempts to differ from others. Instead of focusing on how to deliberately improve the existing work expansion on option pricing model, this research concentrates on exploiting the option-implied information with specific objective on assessing its performance. This serves a crucial line, especially in dealing with portfolio selection problems.

Optimizing or selecting portfolio with optimal wealth allocation has been well acknowledged as a typical classic issue faced by investors. The theoretical study on improving portfolio selection has been the main focus of many researchers. This is obvious especially after the seminal study done by Markowitz (1952). The fact that option information is proven to efficiently encapsulate derivative market perception has triggered many others on studying the optimal selection of a portfolio by exploiting the option moments. A wide spectrum of study tends to utilise historical return data in estimating the option moments. However the portfolio that is based on historical-data estimation has been found to be poorly performed out-of-sample (DeMiguel et al. (2009)). Echoing to this concern, this research utilises option moments implied by option prices, rather than focusing on the use of historical data in improving option moments estimated in constructing an optimal portfolio strategy.

Option-implied information is inferred from the option prices, hitherto is referred as forward-looking option-implied moments. This approach can be perceived as an alternative to the backward-looking historical data. Owing to its forward-looking nature, these option-implied moments are able to comprehensively capture the derivative market perception better than that of the historical data (See Kempf et al. (2014)). It is then expected that the estimation done based on these forward-looking implied moments to perform superiorly in constructing an optimal portfolio. There are several aspects of study on the option-implied moments used in selecting portfolios. One can either consider option-implied volatility, correlation, skewness, risk premium, beta or covariance. This is evident in a plethora of empirical studies that estimate option-implied moments in a number of ways (Kostakis et al. (2011), Aït-Sahalia and Brandt (2008), and DeMiguel et al. (2013)). This is indeed a fertile ground that offer promising avenue for further exploration in which this study attempts to fill into.

The availability of data includes the different step size of strike prices. Since the strike prices range is not continuous, this leads to discretization errors due to numerical integration (Jiang and Tian (2005)). The bias can be induced by the different discreteness of strike prices . Realising that, this research differentiates itself from other existing literature by examining the performance of how a model-free volatility (MFV) is approximated against the different strike price discreteness. This study investigates the respective relationship which governs the selection of strike price step size that leads to the least error. The estimation of the option-implied moments based on the different discreteness of strike price is based on two core strands of literature, i.e. Bakshi et al. (2003) and Buss and Vilkov (2012). The volatility implied by the BSM option pricing model is set as the point of reference value.

This study intends to empirically investigate the index options data, specifically those that are able to directly proxy the global index options market. For that reason, the Dow Jones Industrial Average (DJIA) index options data is utilised in this study. DJIA is the most cited and the most extensively accepted stock market indices. The sample data considered in this study covers the period from January 2009 until the end of 2015. The overarching of this study generally focused on examining whether the different step size of strike price discreteness contributes to the performance of a model-free variance in approximating the real-value volatility. The volatility is proxied by the volatility implied by the Black-Scholes-Merton option pricing model.

This paper is divided into a number of sections. A brief background of study is provided in the first section. The data utilised in this paper is illustrated in Section 2. Section 3 presents the methodology used in assessing the performance of Model-free Bakshi-Kapadia-Madan (MFBKM). The main findings of this study are presented in Section 4. Finally, we conclude in Section 5.

2. Data

This paper utilises all call and put options on the Dow Jones Industrial Index (DJIA) traded daily on the Chicago Board Options Exchange (CBOE) during the period of January 2009 until December 2015. The daily index data retrieved from the DJIA are composed of trading date, expiration date, closing price, exercise price and trading volume for each trading option. The underlying price used in this study will utilise the closing price of the DJIA index, whereas the actual option price is taken from the closing price of the option price. In this study, we utilise the Dow Jones Industrial Average (DJIA) index options data. The options consists of the 30-blue chipped companies index and equity options which represent the most heavily traded and listed in US.

3. Methodology

In order to investigate which selection of strike price step size that leads to the smallest approximation error, seven different discreteness of strike prices are considered. Each strike price step size is then used in estimating the option-implied moment. Generally, this study relies on two core strands of literature, i.e. Bakshi et al. (2003) and Buss and Vilkov (2012). The approaches used in the two studies are mainly adopted in this research with several adjustments and modifications. In order to obtain the option-implied moments values, we adopt the same methodology as in Buss and Vilkov (2012), which is from the estimated moments of the market index return.

However, instead of considering all moments, this study focuses on the variance contract, i.e. the model-free variance (MFV). For the sake of examining the performance of how the model-free volatility is approximated against the different strike price discreteness, this study considers strike step discreteness of \$0.50, \$1.00, \$2.00, \$2.50, \$4.00, \$5.00, and \$10.00. The approximation error is calculated based on how the square-root of model-free variance approximates the volatility estimated using the Black-Scholes-Merton (BSM) model. The volatility implied by the BSM option pricing model is utilised as the benchmark value. The best-performed strike price step size is depicted by the least error induced by the approximation.

3.1 Model-Free Bakshi-Kapadia-Madan

We calculate the option-implied moments based on the extraction approach introduced in Bakshi et al. (2003). The moments include variance contract, cubic contract, quartic contract, model-free implied volatility, as well as model-free option implied skewness. We take into account the model-free framework since the whole information of the BSM implied volatility smile can be considered using this model. Moreover, this model outperforms the BSM volatility in foreseeing realized volatility.

We first compute the option-implied higher moments from the market index data using the same methodology utilised in Bakshi et al. (2003). However, the theoretical foundation behind these model-free higher moments is beyond our scope. We, therefore, will not discuss it in this paper. The respective computation of option-implied moments, as derived by Bakshi et al. (2003) are as follows:

$$R(t,T) \equiv \ln S(t+T) - \ln S(t); \tag{1}$$

$$V(t,T) \equiv E_t^* \left\{ e^{-rt} R(t,T)^2 \right\}; \tag{2}$$

$$W(t,T) \equiv E_t^* \left\{ e^{-rt} R(t,T)^3 \right\}; \tag{3}$$

$$X(t,T) \equiv E_t^* \left\{ e^{-rt} R(t,T)^4 \right\}. \tag{4}$$

Equations (1) and (2) represent the variance contract, denoted as V. The cubic contract is depicted by Equation (3) as W; while Equation (4) represents the quartic contract which is signified by X. The model-free option-implied volatility (MFIV) is simply the square root of Equation (2):

$$MFIV(t,T) = \sqrt{V(t,T)}. \tag{5}$$

Let $S(t)$ be the stock price at time t, r be the risk-free interest rate, $K(t)$ be the strike price at time t, and $R(t,T)$ be the T-log return. $C(t)$ and $P(t)$ are the price of call and put option, respectively, at time t. The model-free option-implied skewness (MFIS) is obtained based from Equations (1) to (4).

$$MFIS(t,T) = \frac{e^{rt}W(t,T) - 3\mu(t,T)e^{rt}V(t,T) + 2(\mu(t,T))^3}{(e^{rt}V(t,T) - (\mu(t,T))^2)^{3/2}}. \tag{6}$$

Besides, Bakshi, Kapadia, and Madan (2003) show that the three defined contracts can attain the following forms:

$$\begin{aligned} V(t,\tau) = & \int_{S_t}^{\infty} \frac{2\left(1 - ln\left[\frac{K}{S_t}\right]\right)}{K^2} C(t,\tau;K)dK \\ & + \int_{0}^{S_t} \frac{2\left(1 - ln\left[\frac{K}{S_t}\right]\right)}{K^2} P(t,\tau;K)dK; \end{aligned} \tag{7}$$

$$\begin{aligned} W(t,\tau) = & \int_{S_t}^{\infty} \frac{6ln\left[\frac{K}{S_t}\right] - 3\left(ln\left[\frac{K}{S_t}\right]\right)^2}{K^2} C(t,\tau;K)dK \\ & - \int_{0}^{S_t} \frac{6ln\left[\frac{K}{S_t}\right] - 3\left(ln\left[\frac{K}{S_t}\right]\right)^2}{K^2} P(t,\tau;K)dK; \end{aligned} \tag{8}$$

$$\begin{aligned} X(t,\tau) = & \int_{S_t}^{\infty} \frac{12\left(ln\left[\frac{K}{S_t}\right]\right)^2 - 4\left(ln\left[\frac{K}{S_t}\right]\right)^3}{K^2} C(t,\tau;K)dK \\ & + \int_{0}^{S_t} \frac{12\left(ln\left[\frac{K}{S_t}\right]\right)^2 - 4\left(ln\left[\frac{K}{S_t}\right]\right)^3}{K^2} P(t,\tau;K)dK. \end{aligned} \tag{9}$$

The risk-neutral variance is depicted as:

$$VAR(t,\tau) \equiv E^q\left\{(R_{t,\tau} - E^q\left[R_{t,\tau}\right])^2\right\} \tag{10}$$

;

$$VAR(t,\tau) = e^{r\tau}V(t,\tau) - \mu(t,\tau)^2. \tag{11}$$

Recall that in Equation (6) the risk-neutral skewness is shown as

$$\begin{aligned} & MFIS(t,\tau) \\ & \equiv \frac{E^q\left\{R_{t,\tau} - E^q[R_{t,\tau}]^3\right\}}{E^q\left\{R_{t,\tau} - E^q[R_{t,\tau}]^2\right\}^{3/2}} \\ & = \frac{e^{r\tau}W(t,\tau) - 3e^{r\tau}\mu(t,\tau)V(t,\tau) + 2\mu\left(t,\tau\right)^3}{\left[e^{r\tau}V(t,\tau) - \mu\left(t,\tau\right)^2\right]^{3/2}}. \end{aligned} \tag{12}$$

Whereas, the risk-neutral kurtosis is as follows:

$$MFIK(t,\tau) \equiv \frac{E^q\left\{(R_{t,\tau} - E^q[R_{t,\tau}])^4\right\}}{E^q\left\{(R_{t,\tau} - E^q[R_{t,\tau}])^2\right\}^2}; \tag{13}$$

$$\begin{aligned} MFIK(t,\tau) = & \frac{e^{r\tau}X(t,\tau) - 4e^{r\tau}\mu(t,\tau)W(t,\tau)}{\left[e^{r\tau}V(t,\tau) - \mu(t,\tau)^2\right]^2} \\ & + \frac{6e^{r\tau}\mu(t,\tau)^2 V(t,\tau) - 3\mu(t,\tau)^4}{\left[e^{r\tau}V(t,\tau) - \mu(t,\tau)^2\right]^2}, \end{aligned} \tag{14}$$

in which μ-expectation is

$$\mu(t,\tau) = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V(t,\tau) - \frac{e^{r\tau}}{6}W(t,\tau) - \frac{e^{r\tau}}{24}X(t,\tau). \tag{15}$$

Recall that Equations (1) and (2) are simply representing the variance contract. The cubic contract is depicted by Equation (3); while Equation (4) represents the quartic contract. This study focuses on the variance contract, i.e. the model-free variance (MFV).

4. Results and Discussions

In this section, the performance of how the model-free volatility is approximated against the different strike price discreteness is compared. The volatility estimated using the BSM option pricing model is placed as the point of reference. By hypothesis, the strike price step having the smallest discreteness is believed to deliver smaller approximation error compared to other strike step considered.

For better illustration, this study considers strike step discreteness of \$0.50, \$1.00, \$2.00, \$2.50, \$4.00, \$5.00, and \$10.00. The approximation error is calculated subject on how the square-root of model-free variance approximates the volatility estimated using the BSM model. The approximation errors as the function of different strike price discreteness are tabulated for both call and put options. The respective results are reported in Table 1 and Table 3.

Consistent findings are observed in both types of options, in which the strike price step of \$0.50 is recorded to deliver the smallest approximation error. More to the point, the biggest approximation error is contributed by strike step of \$10.00. The strike price discreteness size is found to have linear relationship with the approximation error. Again, this supports the hypothesis of this study in both cases. The results are further verified in Figure 1 and Figure 2, respective to the call and put options. A linear line is evident in both figures, depicting a linear relationship between the strike price discreteness size and the approximation error.

Table 1: Approximation Error of the Different Strike Step Size for Call Options

Strike Step (\$)	MFV	MFIV	Approximation Error (%)
0.50	0.0473	0.2174	-3.361
1.00	0.0492	0.2218	-1.402
2.00	0.0522	0.2284	1.526
2.50	0.0531	0.2304	2.417
4.00	0.0584	0.2416	7.372
5.00	0.0609	0.2468	9.689
10.00	0.0768	0.2771	23.174

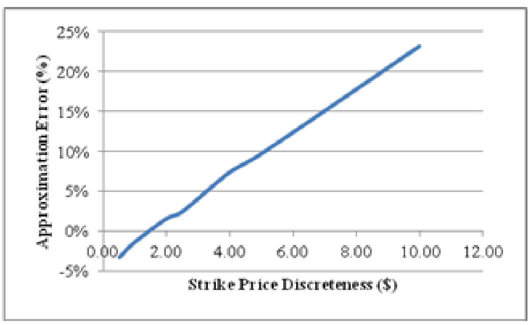

Figure 1: Approximation Error versus Strike Price Discreteness for Call Options

Table 2: Goodness-of-Fit Analysis for Call Options

Model	f(x) = p1*x + p2
Coefficients (with 95% confidence bounds):	
p1	p2
0.02767 (0.02642, 0.02892)	-0.04252 (-0.04837, -0.03667)
Goodness of Fit:	
SSE	7.51x10-5
R-Square	0.9985
Adjusted R-Square	0.9981
RMSE	3.876x10-3

Table 3: Approximation Error of the Different Strike Step Size for Put Options

Strike Step ($)	MFV	MFIV	Approximation Error (%)
0.50	0.0498	0.2231	-0.824
1.00	0.0514	0.2266	0.716
2.00	0.0545	0.2334	3.712
2.50	0.0569	0.2386	6.029
4.00	0.0609	0.2468	9.677
5.00	0.0656	0.2562	13.857
10.00	0.0825	0.2872	27.646

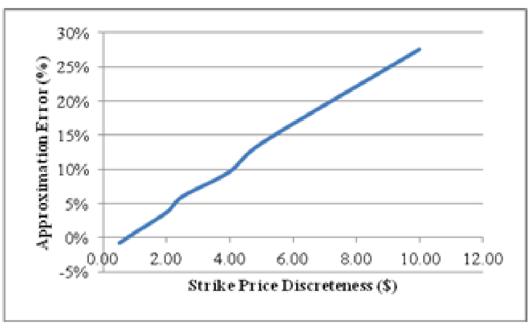

Figure 2: Approximation Error versus Strike Price Discreteness for Put Options

Note that MFV denotes the model-free variance. Hence the square-root of MFV represents the MFIV. The approximation error is calculated based on the relative percentage error.Based on the results of both call and put options, a clear line of conclusion can be established that there exists a linear relationship between strike price discreteness size and approximation error. This claim is further supported by the curve-fit analysis as presented in Figure 3 and Figure 4, respectively for call and put options.

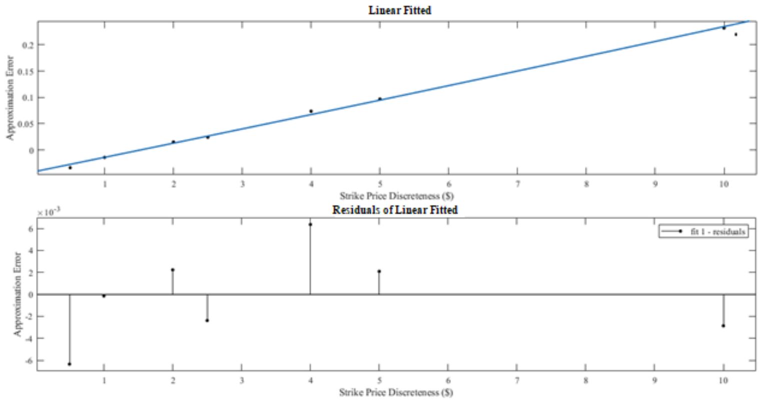

Figure 3: Linear Fitted of Strike Price Discreteness Accuracy for Call Options

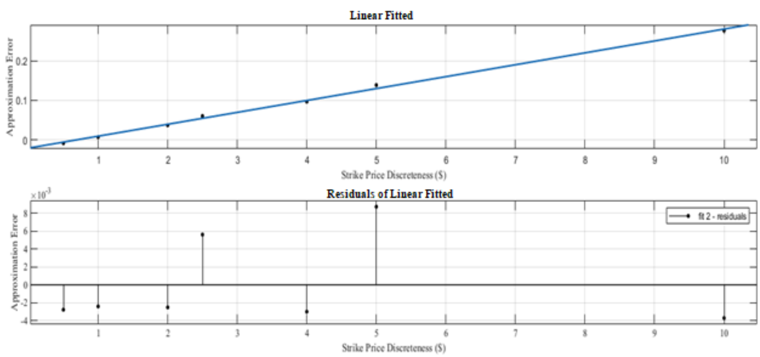

Figure 4: Linear Fitted of Strike Price Discreteness Accuracy for Put Options

The goodness-of-fit analysis of the fitted line is reported in Table 2 and Table 4, respectively. The fitted line is best explained by linear model polynomial of first order, replicated by $f(x) = p1x + p2$. 99 percent of adjusted R-square is recorded in both cases of fitting, suggesting that the strike price discreteness and approximation error relationship is best explained by the linear model of polynomial of order one. This advocates that the best strike step to be chosen theoretically should be of that \$0.50. However, since the results are subjected to data availability, especially that of greater than \$100, the most suitable and practical strike price step size that asserts the most minimum approximation error is of \$1.00.

Table 4: Goodness-of-Fit Analysis for Put Options

Model	f(x) = p1*x + p2
Coefficients (with 95% confidence bounds):	
p1	p2
0.03007 (0.02829, 0.03184)	-0.0205 (-0.02878, -0.01223)
Goodness of Fit:	
SSE	1.50x10-4
R-Square	0.9974
Adjusted R-Square	0.9968
RMSE	0.005485

5. Conclusions

This research differentiates itself from other existing literature by investigating the performance of how the model-free volatility (MFV) is approximated against the different strike price discreteness. This study focuses on studying the trend that rules behind the function of approximation error against the strike price step. Steady findings are obtained for both call and put options affirm that there exists a pattern of linear line governing between the strike price discreteness size and approximation error. The best performed strike price discreteness size having the smallest approximation error is applied to that of \$0.50, theoretically. However, the use of strike step of \$0.50 is quite impractical due to the inefficient volume and availability of data recorded for strike size more than \$1.00 as for this case. Thus, the most appropriate and convenient strike price step size that asserts the most minimum approximation error is of \$1.00. The choice of the different step size of strike price discreteness is important in contributing to the performance of a model-free variance in approximating the real-value volatility.

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